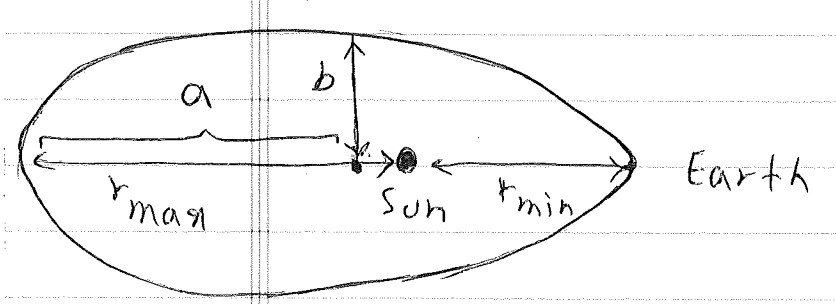
Lec 1: 08/24/2010

Introduction:

In this course the focus will be on the stars and stellar evolution; how stars form and how they die.

Lets start with some information about the nearest star to us, Sun, and see whether we can make sense of that through some rough estimates. This will also help us understand some general facts about the stars.

Earth orbits around Sun in an elliptical orbit (according to Newton's gravitational law):



Prihelion: $r_{min} = 1.4710 \times 10^{13}$ cm

Aphelion: $r_{max} = 1.5210 \times 10^{13}$ cm

The semi-major axis of the ellipse is called an "Astronomical Unit":

1 A.U. $\equiv a = 1.4960 \times 10^{13}$ cm

Eccentricity of the ellipse is defined as:

$$e \equiv \frac{\sqrt{a^2 - b^2}}{a} = 0.0167$$

$e = 0$ corresponds to a circle, hence the orbit is very close to a circle.

Solar radius is:

$$\boxed{R_\odot = 6.9 \times 10^{10} \text{ cm} \simeq 100 R_E}$$

The solar mass can be found from Newton's 2nd law (assuming a circular orbit):

$$\frac{m_E V_E^2}{\underbrace{R_{E-S}}_{R_{E-S} \simeq a}} = \frac{G M_\odot m_E}{R_{E-S}^2} \Rightarrow \boxed{M_\odot = 1.97 \times 10^{33} \text{ g}} \quad \left(V_E = 2.98 \times 10^6 \frac{\text{cm}}{\text{s}}\right)$$

Essentially, the mass of observed stars lie within the range $(0.1 - 60) M_\odot$. This is something we would like to understand.

Knowing solar mass and radius, we can find mean solar density:

$$\bar{\rho}_\odot = \frac{M_\odot}{\frac{4}{3}\pi R_\odot^3} \sim 1.4 \ g/cm^3$$

Also, acceleration of gravity at solar surface is:

$$g_\odot = \frac{G M_\odot}{R_\odot^2} \sim 2.7 \times 10^4 \ \frac{cm}{s^2} \sim 27 g$$

At the center ($r=0$) we have:

$$\boxed{\rho_\odot \simeq 150 \ g/cm^3 \ , \ T_\odot \sim 1.5 \times 10^7 \ K}$$

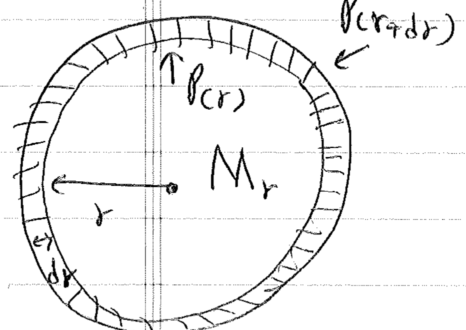
The very high temperature (~1 keV in natural units) implies that Sun is gaseous with an equation of state that to a very good approximation is that of a perfect gas. In fact, the interior is hot enough for everything (~70% Hydrogen, ~26% Helium) to get ionized:

I.P. : H = 13.6 eV He = 24.76 eV He^+ = 54.4 eV

We would like to understand why the Sun is stable and what the source of Sun energy is.

Hydrostatic Equilibrium:

Consider a spherically symmetric configuration of gas. Gravity results in contraction while pressure opposes contraction. In case of hydrostatic equilibrium forces from gravity and pressure balance each other:



$$\frac{\partial P_{(r)}}{\partial r} + \frac{G M_r}{r^2} \rho_{(r)} = 0 \Rightarrow$$

$$\boxed{\frac{\partial P_{(r)}}{\partial M_r} = -\frac{G M_r}{4\pi r^4}}$$

This is the equation for hydrostatic equilibrium, where M_r is the mass contained within radius "r". Note that $P_{(r)}$ is a monotonically decreasing function of M_r, hence r, in hydrostatic equilibrium, and $P=0$ at surface.

The total energy of the system is:

$W = U + \Omega$ (U: Internal energy, Ω: Gravitational potential energy)

$$\Omega = -\int_0^M \frac{G M_r}{r}\, dM_r = \boxed{-\frac{q G M^2}{R}} \quad *$$

Here M is the total mass, R is the radius, and q is a constant. One can show that $q = \frac{3}{5}$ for a constant density.

In hydrostatic equilibrium we have (according to Virial theorem):

$2K + \Omega = 0$ (K: Kinetic energy)

For an isotropic gas pressure P follows:

$$P = \frac{1}{3}\int n(\vec{p})\, \vec{p}\cdot\vec{v}\, d^3p \implies 2K = 3\int_V P\, dV$$

Thus:

$$\int \frac{3P}{\rho}\, dM_r + \Omega = 0$$

Equation of state relates pressure P to energy density ρu (u being internal energy per unit mass):

$P = (\gamma - 1)\rho u$ (perfect gas)

For example, for a gas of photons $\gamma = \frac{4}{3}$ and for a monatomic (non-relativistic) gas $\gamma = \frac{5}{3}$.

Assuming the only contribution to the internal energy comes from kinetic energy, we find:

$$3(\gamma-1)U+\Omega=0 \Rightarrow \boxed{W=\frac{3\gamma-4}{3(\gamma-1)}\Omega} \quad **$$

For $\gamma>\frac{4}{3}$ total energy is negative. Therefore, it will decrease as a result of contraction. However, conservation of energy requires that the change in W be transferred outside:

$$L=-\frac{dW}{dt} \quad (L: \text{Luminosity})$$

Using equations *, ** we find:

$$L=-\frac{9}{2}\frac{GM^2}{R}\left(\frac{\dot{R}}{R}\right)$$

The characteristic time scale for contraction is called Kelvin-Helmholtz time scale:

$$t_{KH} \approx \frac{9}{2}\frac{GM^2}{LR} \quad (\Delta R \approx \dot{R}t_{KH} \sim R)$$

Plugging in numerical values of M, R, L for Sun we obtain:

$$\boxed{t_{KH}^{Sun} \approx 2 \times 10^{7} \ yr}$$

Therefore, if Sun energy came from contraction alone (equivalently, there was no other compenent of internal energy other than kinetic energy), its radius would significantly change over 20 million years. Such a change would affect Earth and life on Earth. However, there is no such evidence from fossil records.

This simple calculation shows that Sun is powered by something more than gravitational contraction. Namely, nuclear burning as we will see later.

<u>Simple Estimates of T, ρ at the Sun Center:</u>

Next, lets make simple estimates of T, P at the center of Sun. Again, we use perfect gas approximation. Recall the equation for hydrostatic equilibrium:

$$\frac{\partial P}{\partial r} = -\frac{G M_r}{4\pi r^4}$$

We approximate $\frac{\partial P}{\partial r}$ by $\frac{\Delta P}{\Delta r}$, where:

$\Delta P = P_0 - P_c = -P_c$ ($P_0 = 0$; pressure at surface, P_c: pressure at center)

$\Delta r = R_\odot$

After replacing M_r and r by rough mean values $\frac{M}{2}$, $\frac{R}{2}$ we find:

$$P_c \simeq \frac{2 G M_\odot}{\pi R_\odot^4}$$

Considering a proton-electron gas (from ionization of Hydrogen, we neglect Helium), we have:

$P = n \mathcal{R} T$

Where n is the number density of particles (twice the number density of protons, because of neutrality) and

$\mathcal{R} = 8.315 \times 10^{7}$ erg $K^{-1} g^{-1}$. For a monatomic gas we have:

$P = \frac{2}{3} \rho E$ ($\rho = \nu n$, ν: mean molecular weight)

Where $\nu = 0.5$ for Hydrogen. This can be understood as both electron and proton make (equal) contributions to the pressure, while the dominant contribution to mass density comes from proton.

Putting everything together, we obtain:

$$T_c = \frac{P_c}{\rho_c} \frac{\nu}{\mathcal{R}} = P_c \frac{\nu}{\mathcal{R}} \frac{\bar{\rho}_\odot}{\rho_c} \frac{4\pi R_\odot^3}{3 M_\odot} \simeq \frac{8}{3} \frac{\nu}{\mathcal{R}} \frac{G M_\odot}{R_\odot} \frac{\bar{\rho}_\odot}{\rho_c}$$

Since $\bar{\rho}_\odot < \rho_c$, we get an upper bound on T_c:

$$\boxed{T_c \lesssim \frac{8}{3} \frac{G \nu}{\mathcal{R}} \frac{M_\odot}{R_\odot} = 3 \times 10^{7} K}$$

This is in good agreement with the value $\simeq 1.5 \times 10^{7}$ K that was mentioned earlier. For pressure we find an estimated value:

$$P_c \approx 7 \times 10^{15} \frac{dyn}{cm^2}$$

This is an order of magnitude below that from numerical simulations $\approx 2.7 \times 10^{17} \frac{dyn}{cm^2}$. The estimated value can be improved by a more careful treatment of sun composition.